

## Elastic bottom propagation mechanisms investigated by parabolic equation methods

Scott D. Frank

Marist College

Department of Mathematics

Poughkeepsie, NY 12601

phone: (845) 575-3977 fax: (845) 575-3605 email: [scott.frank@marist.edu](mailto:scott.frank@marist.edu)

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### LONG-TERM GOALS

Develop elastic parabolic equation (PE) method capabilities in order to study elastic propagation mechanisms and their effects on underwater acoustic environments in the form of scattering at an elastic interface, oceanic  $T$ -waves, and Scholte waves.

### OBJECTIVES

To implement explosive and earthquake type seismic sources in current elastic parabolic equation methods for underwater acoustic environments. These will be used to investigate propagation mechanisms that depend on the elastic properties of the ocean bottom and result in acoustic propagation in the ocean. Two examples of such propagation are oceanic  $T$ -waves, which are acoustic waves that result from earthquake or buried explosive sources, and Rayleigh-type waves along the ocean floor, whose existence requires horizontal and vertical displacements present in elastic sediments. The generation, propagation, and potential contribution of these waves to the otherwise quiet acoustic field of the deep ocean all require study, in particular as potential sources of unexplained deep shadow zone arrivals that have been experimentally observed below the ray-theoretic turning point.[1, 2]

### APPROACH

In a cylindrically symmetric environment, where  $r$  is the distance from the source and  $z$  is depth, recent parabolic equation methods for acoustic propagation in elastic sediments are based on the  $(u_r, w)$  formulation of elasticity.[3] In this formulation  $u_r$  is the horizontal derivative of the horizontal displacement and  $w$  is the vertical displacement. The outgoing portion of the separated Helmholtz operator results in the parabolic equation solution for a range-independent environment,

$$\frac{\partial}{\partial r} \begin{pmatrix} u_r \\ w \end{pmatrix} = i(L^{-1}M)^{1/2} \begin{pmatrix} u_r \\ w \end{pmatrix}, \quad u_r = \frac{\partial u}{\partial r}, \quad (1)$$

where  $L$  and  $M$  are matrices containing depth-dependent operators that incorporate compressional wave speed,  $c_p$ , shear wave speed,  $c_s$  and density  $\rho$  via the Lamé parameters of the elastic medium,  $\lambda$  and

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$\mu$ .[3] Range dependence is included by modeling sloping boundaries with a stair-step approximation and applying appropriate matching conditions at each vertical interface. This formulation is computationally stable and is combined with rotated variable methods[4] and an elastic single-scattering approximation[5] to improve accuracy for complex multi-layered range-dependent underwater acoustic environments.[6] Parabolic equation self-starters have been used for sources in a fluid[7] and for sources in an elastic material without an overlying fluid layer[3], and more recently were benchmarked in underwater acoustic environments.[8]

Wavenumber content of range independent portions of an elastic PE solution  $f(r)$  is calculated using the Hankel transform

$$F(k,z) = \int_{r_0}^R f(r,z) J_0(kr) r dr \quad (2)$$

where  $r_0$  and  $R$  mark the beginning and end of a range-independent portion of the environment, and  $J_0(kr)$  represents the Bessel function of the first kind of order 0.

There are two primary mechanisms of oceanic  $T$ -wave generation.[9] Downslope conversion occurs when elastic waves interact with a sloping ocean bottom and the resulting transmission angle focuses acoustic waves into the SOFAR channel. Abyssal  $T$ -waves are generated in the deep ocean where there is no sloping bottom. It is believed that these are caused by ocean bottom roughness scattering the elastic waves up into the water column.[10] Characteristics of underwater acoustic fields in the presence of randomly generated Normally distributed ocean bottom roughness are determined by analysis of transmission loss results and wavenumber spectra resulting from multiple realizations of these environments.

Scholte interface waves are known to be excited by seismic sources and have been observed by seismometers at the ocean bottom.[11, 12] Energy from interface waves has also been detected by hydrophones near the seafloor[13] well below the SOFAR channel, suggesting that these waves could influence deep-shadow zone arrivals observed during NPAL.[1, 2] Elastic wave theory predicts that the Scholte wave speed is approximately  $0.8c_s$ . Thus, a peak in the wavenumber spectra calculated from elastic PE solutions near this speed indicates interface waves have been excited.

## WORK COMPLETED

- The capability of PE solutions to generate oceanic  $T$ -waves via downslope conversion has been verified by comparing transmission loss results for flat and range-dependent downslope environments. In particular, depth-averaged transmission loss for downslope environments is several dB lower than for flat environments. These results occur both for purely shear and purely compressional seismic source self-starters.[13]
- The impact of elastic parameters such as compressional wave speed, shear wave speed, and material density on downslope conversion of elastic waves into oceanic  $T$ -waves has been characterized. For example effects of downslope conversion appear to increase as  $c_p$  increases for ocean bottoms composed of a Poisson solid.
- Range-dependence in the form of ocean bottom roughness has been shown to generate abyssal  $T$ -waves in PE solutions with seismic self-starters.[13, 14]
- Broadband calculations in the 5-20 Hz frequency range appear to effectively model  $T$ -wave

arrivals observed at ocean hydrophone arrays. These solutions should allow the elastic PE to be useful in comprehensive test ban treaty monitoring.

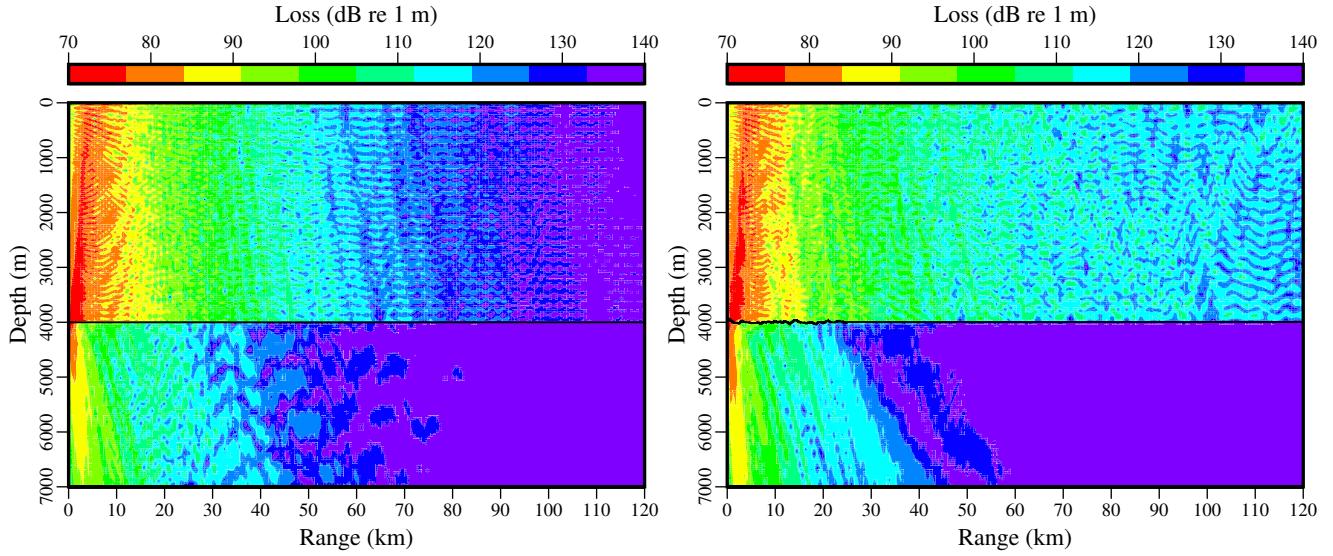
- Elastic PE solutions for acoustic sources in range-dependent elastic bottom environments contain Scholte interface waves at ranges greater than 100 km.
- Elastic PE solutions and elastic coupled mode results have been shown to generate comparable Scholte interface waves for range dependent environments.[15]
- Hankel transform methods characterize how large ocean bottom topography leads to near bottom acoustic fields for ranges greater than 100 km and water column depths greater than 2 km.

## RESULTS

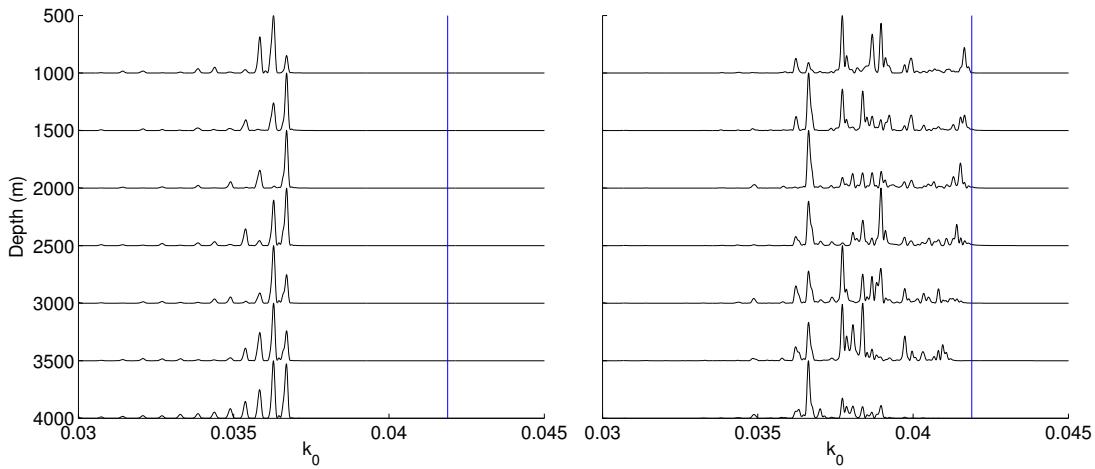
Physical theories suggesting that abyssal  $T$ -waves are generated by range-dependence of the ocean bottom are confirmed with elastic PE solutions. Figure 1 compares transmission loss resulting from a 6 km deep (2 km below ocean bottom) 10 Hz seismic source. The ocean sound speed is modeled with a Munk deep water profile. The left panel shows a 4 km deep ocean with a range-independent bottom. There is more than 140 dB of loss at 120 km range throughout the water column. High order modes are indicated by steep angle beams that result in multiple interactions with the ocean surface and ocean bottom. The right panel shows nearly the same environment, but randomly generated range-dependence has been incorporated into the first 25 km of the ocean bottom. At 120 km range there is nearly 30 dB less loss than in the range independent case. In addition, lower order propagating acoustic modes are apparent near the sound channel axis of the water column.

Hankel transforms of these acoustic fields are shown in Fig. 2. The left panel corresponds to wavenumber spectra for every 500 m depth from 1000 to 4000 m for the range-independent environment on the left of Fig. 1. Peak values only occur for high-order acoustic modes that propagate at high angles in the water column resulting in multiple interactions with the ocean bottom and substantial attenuation. The right panel corresponds to wavenumber spectra at the same depths for the range-dependent random bottom environment on the right of Fig. 1. Lower order acoustic modes are evident and result in the demonstrated longer range acoustic propagation.

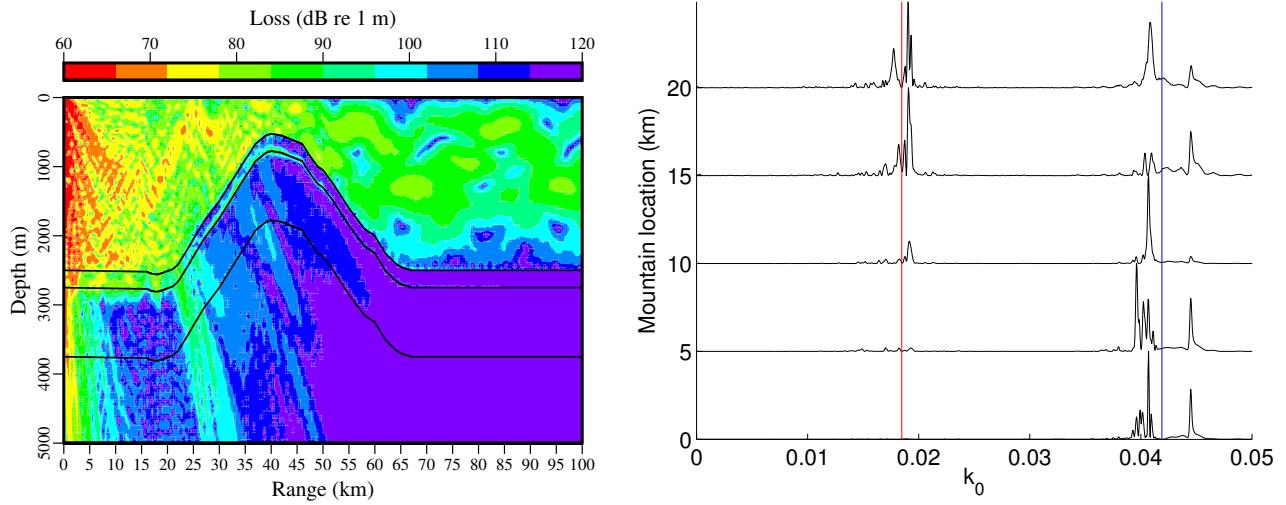
The left panel of Fig. 3 shows an elastic PE solution for a 500 m deep 10 Hz acoustic source in a 2.5 km deep ocean environment with a layered elastic bottom and an intervening seamount. Range-dependence associated with the seamount begins 15 km from the source. Acoustic wave energy interacts with the elastic layers. Channeling of elastic energy in the top elastic layer appears on the left side of the seamount and continues over the seamount's peak. The presence of a Scholte interface wave is indicated by fingers of reduced transmission loss in the water column near the ocean bottom on the right side of the seamount, in particular at ranges greater than 65 km. Wavenumber spectra for a depth of 2550 m obtained from environments with the seamount starting at ranges of 0, 5, 10, 15, and 20 km from the source are shown in the right panel of Fig. 3 (the left panel corresponds to a 15 km seamount location). The blue vertical line represents 1500 m/s. Red vertical line is associated with compressional speed in top elastic layer. Wavenumber peaks to the right of the blue line are consistent with wave speeds associated with Scholte interface waves. The variation of these peaks with range indicates that the location of the seamount impacts whether or not these arrivals can be observed. This suggests that Scholte interface waves could explain observed deep sea floor arrivals, even from an acoustic source, for certain configurations of intervening ocean bottom topography.



**Figure 1:** Elastic parabolic equation solutions with a purely shear 10 Hz seismic self-starter demonstrate the generation of abyssal T waves by ocean bottom roughness. In the left panel the ocean-sediment interface is perfectly flat over 120 km. High order modes are indicated by the diagonal up and down pattern characteristic of ocean surface and ocean bottom reflected energy. Transmission loss after 120 km is over 140 dB. The right-hand panel includes randomly generated bottom roughness over the first 25 km. Substantial energy remains (nearly 20 dB less transmission loss) even at 120 km range. The rough interface has scattered the energy into lower order modes that propagate in the middle of the water column.



**Figure 2:** Wavenumber spectra obtained from Hankel transform over range-independent portions of environments in Fig. 1 every 500 m of depth from 1000 to 4000 m. The left panel corresponds to the range independent environment in Fig. 1 and shows slowly propagating high order modes. These modes suffer multiple interactions with the ocean bottom leading to large transmission loss results. Right hand panel shows that portions of the acoustic field have been scattered into lower order modes that are further to the right on the wavenumber spectrum near the blue line that corresponds to 1500 m/s. These modes result in long range propagation that is recorded as T-waves.



**Figure 3:** Deep acoustic arrivals and Scholte type interface waves appear in elastic PE solution for a layered range-dependent seamount environment. Left panel is a contour plot of transmission loss results from the elastic PE with a 10 Hz acoustic source located 500 m below the ocean surface. The seamount occurs in a 2500 m deep ocean environment with elastic layers. Small fingers of reduced transmission loss near the ocean bottom at ranges greater than 60 km represent acoustic energy in the ocean resulting from interface wave propagation. Right panel shows wavenumber spectra at 2450 m depth obtained by calculating the Hankel transform over the range independent portion to the right of the seamount. Curves are shown for environments with the start of the seamount at 0, 5, 10, 15, and 20 km away from the acoustic source. Spectra show clear low order propagating acoustic modes to the left of the blue line, which corresponds to 1500 m/s. Wavenumber peaks to the right of the blue line arrive at expected values for Scholte interface waves. The maximum value of these peaks varies with the range of the seamount, suggesting the feature is more efficient at exciting interface waves in certain configurations.

## IMPACT/APPLICATIONS

- Improved modeling capabilities of elastic parabolic equation methods for underwater acoustic problems where elastic properties of the bottom cannot be ignored. Specifically for seismic sources that are relevant for geophysical study or test ban treaty monitoring.
- Potential explanation for "deep seafloor arrivals" and the reception of acoustic signals in what may be otherwise considered a quiet ocean environment for monitoring.
- Advances in modeling elastic layers have potential application in ice covered environments where an elastic layer lies on top of the water column.

## RELATED PROJECTS

This research relates to the separately funded work of Robert Odom regarding the two-way coupled mode code. It also relates to parabolic equation development by Jon M. Collis (Colorado School of Mines) and Adam Metzler (University of Texas, Applied Research Lab).

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## PUBLICATIONS

- Published in refereed journal (*JASA*): [8]
- Presentations: [13], [14], [15]